Using Manipulatives to Compute With Fractions

It is important that you explore fraction operations using different manipulatives, such as pattern blocks and fraction tiles. The experience will be enhanced if you actually use these manipulatives while solving the problems rather than just reading about them. We've organized the problems by manipulative to facilitate this process. Be sure to solve the problems presented in each task using the indicated manipulative prior to reading the text that follows that task.

Pattern Blocks

It will be helpful for you to explore how pattern blocks can be used to represent fractions if you have not used them in this manner in the past. Pattern blocks represent an area model for fractions and are useful when exploring operations with mixed numbers because the manipulative can represent a number of different quantities.

In this chapter, the hexagon will be used as the whole. Examine what fractional part of the whole would be represented by one triangle, one rhombus (the blue rhombus in the set, not the tan rhombus), or one trapezoid. These are the pieces that will be used in the task that follows. Use pattern blocks to model the problems in figure 1.20. If you do not have easy access to this manipulative, you can find it online at virtual manipulative sites.

Solve with pattern blocks. Use the hexagon as the whole for each problem.

1. $1\frac{1}{3} \div 2 = ?$
2. $\frac{3}{5} \times 2\frac{1}{2} = ?$
3. $3\frac{1}{3} \times \frac{1}{2} = ?$
4. $\frac{5}{6} \div 2 = ?$

Figure 1.20: Fraction operation tasks with pattern blocks.

In the first problem, you likely started with showing $1\frac{1}{3}$ with one hexagon and one rhombus as shown in figure 1.21.

What does $1\frac{1}{3} \div 2$ mean? In this case, division by two can be interpreted as sharing between two groups. This problem could be placed in context by determining how much of a cookie each person would get if $1\frac{1}{3}$ cookies were equally shared between two people. The hexagon and the rhombus could represent $1\frac{1}{3}$ cookies. If the hexagon was split into three rhombuses, there would be four rhombuses in all, so each person would receive two of them. How much of a hexagon is this? Each person would receive $\frac{2}{3}$ of a hexagon, or, with the context provided, $\frac{2}{3}$ of a cookie.

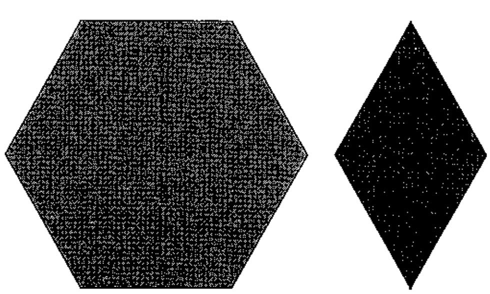

Figure 1.21: Pattern blocks showing $1\frac{1}{3}$.

The second problem, multiplying $\frac{3}{5} \times 2\frac{1}{2}$, may be less straightforward. What does this fraction problem mean? Thinking of it in context will help you find meaning in the

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DO NOW

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2. $\frac{3}{5} \times 2\frac{1}{2} = ?$
3. $3\frac{1}{3} \times \frac{1}{2} = ?$
4. $\frac{5}{6} \div 2 = ?$

Figure 1.20: Fraction operation tasks with pattern blocks.

In the first problem, you likely started with showing 1⅓ with one hexagon and one rhombus as shown in figure 1.21.

What does 1⅓ ÷ 2 mean? In this case, division by two can be interpreted as sharing between two groups. This problem could be placed in context by determining how much of a cookie each person would get if 1⅓ cookies were equally shared between two people. The hexagon and the rhombus could represent 1⅓ cookies. If the hexagon was split into three rhombuses, there would be four rhombuses in all, so each person would receive two of them. How much of a hexagon is this? Each person would receive ⅔ of a hexagon, or, with the context provided, ⅔ of a cookie.

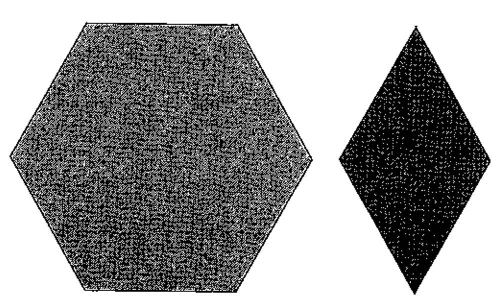

Figure 1.21: Pattern blocks showing 1⅓.

The second problem, multiplying ⅗ × 2½, may be less straightforward. What does this fraction problem mean? Thinking of it in context will help you find meaning in the

operation, which is interesting, because students are often led to believe that word problems make mathematics more difficult. In this instance, they actually make the problem easier to compute with a visual model. What could ⅗ × 2½ represent? It could represent determining how much pie is in ⅗ of 2½ pies. This context helps you realize that the first action is to represent 2½ pies. This is accomplished with two hexagons and one trapezoid. The next action is to split the 2½ pies into five equal parts so that ⅗ of the 2½ can be determined. This may be challenging at first, until you realize that the 2½ can be exchanged for five trapezoids. At this point, the solution to the problem becomes evident. Three-fifths of the five trapezoids is three trapezoids, or 3/2. You can also use two of the trapezoids to make up one hexagon, or one whole, so you have 1½ as the product (see figure 1.22). This answer is reasonable because ⅗ × 2½ is approximately ½ of 3, or 1½.

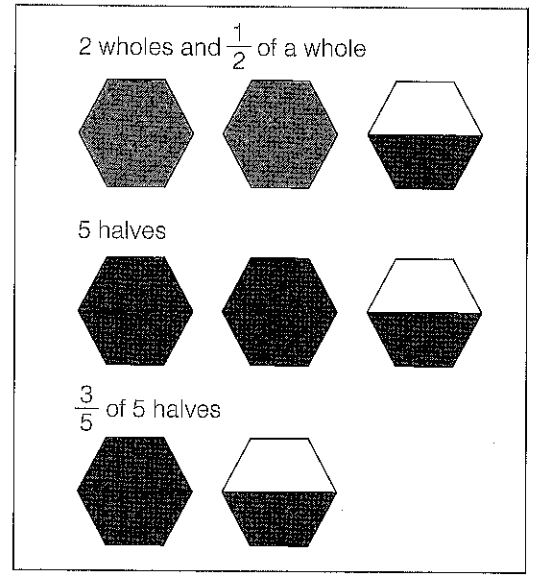

Figure 1.22: Representing ⅗ × 2½ with pattern blocks.

The third problem in figure 1.20 involves finding more than one group of ½. This problem is different from others encountered so far in this chapter. You are finding three groups of ½ and an additional ⅓ of a group of ½, or 3 and ⅓ groups of ½. Three groups of ½ can be represented with pattern blocks using three trapezoids, but how do you find ⅓ of a group of ½? One-third of a group of ½ is found by exchanging a trapezoid for three triangles and using one of them to represent ⅓ of the trapezoid. Figure 1.23 illustrates 3⅓ groups of ½ with three trapezoids and one triangle, which are then combined to make one whole and 4/6 of another whole by exchanging two trapezoids for a hexagon and one trapezoid for three triangles.

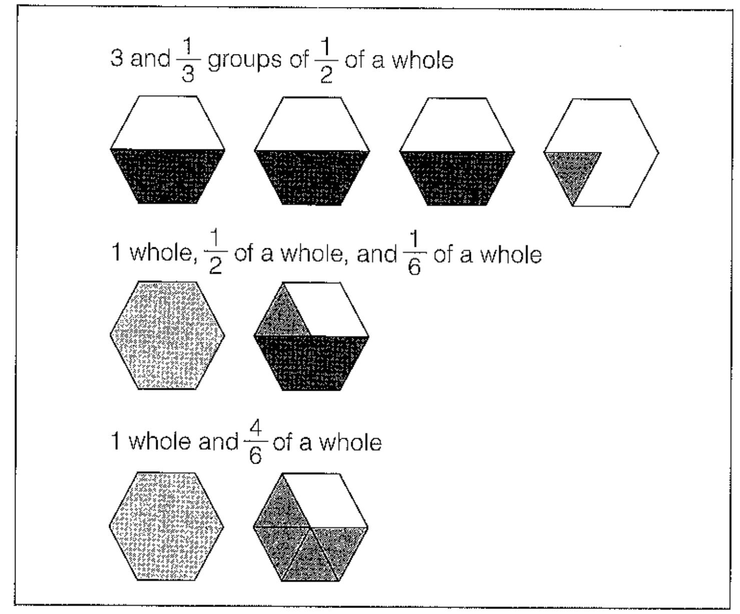

Figure 1.23: Representing 3⅓ × ½ with pattern blocks.

The final problem in figure 1.20 involves division of a fraction by a whole number. To make sense of the problem, it is helpful to represent it in context. What might be a context for this problem? A problem could be something like this:

> Jeff has ⅚ of a pie. How much pie will each person get if he shares it fairly with his friend?

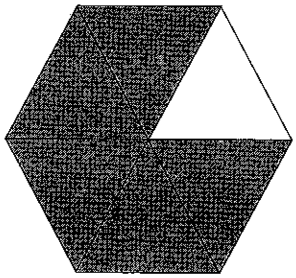

Figure 1.24: Pattern blocks representing 5⁄6.

Once again, the context helps to provide a starting point for representing and solving this problem visually. If the hexagon is your whole, you would begin with five triangles to represent 5⁄6 of the pie (see figure 1.24).

How would you share this fairly between two people? This cannot be modeled with a standard set of pattern blocks. The five sixth-sized pieces, represented by triangles, cannot be shared evenly between two people; there is not a piece that represents half of a sixth. It would seem that this problem could not be solved with pattern blocks, unless the whole was represented by more than one pattern block piece. However, consider this context:

> Jeff has 5⁄6 of a pie. He is supposed to bring 2 pies to the dinner party. What portion of what he is supposed to bring does he have?

Representing the solution process for this problem also begins with five triangles if the hexagon is the whole. Two hexagons represent the amount of pie Jeff needs to bring to the dinner party. The question is answered by determining the fraction of two hexagons covered by the five triangles (see figure 1.25). The answer is 5⁄12 of the two pies. In this context, rather than sharing the 5⁄6 of a pie, the goal was to make a group of two pies; however, there was not enough pie to do this. There was only enough pie to make 5⁄12 of what was needed.

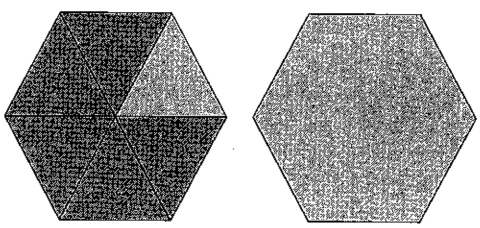

Figure 1.25: Pattern blocks representing 5⁄6 ÷ 2.

A slight change in thinking about the context helps make sense of the problem.

Fraction Tiles

While pattern blocks represent an area model, fraction tiles can be used to represent a linear model. For additional conversation on different fraction models see chapter 3 of *Making Sense of Mathematics for Teaching Grades 3–5* (Dixon et al., 2016). Use fraction tiles to solve the three problems in figure 1.26. These are also available online. Be sure to solve them before reading the discussion of their solutions.

Solve with fraction tiles.

1. $\frac{1}{4} + \frac{2}{3} = ?$
2. $\frac{3}{4} \times \frac{2}{3} = ?$
3. $\frac{5}{12} \div \frac{1}{6} = ?$

Figure 1.26: Fraction operations with fraction tiles task.